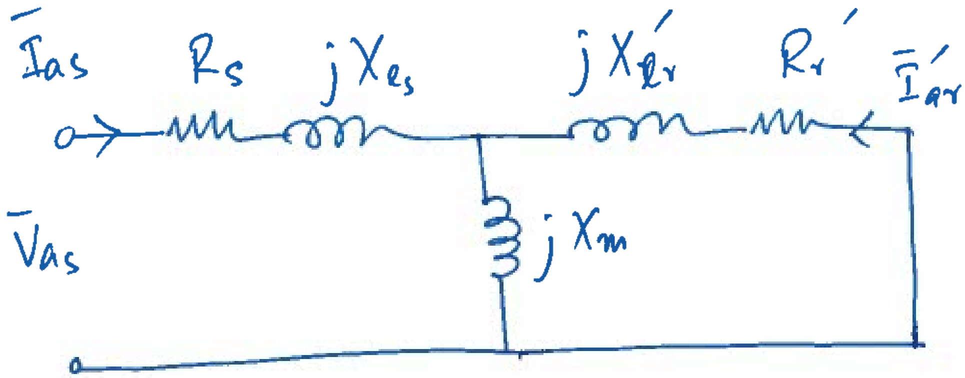


R_s = Resistance of stator

$\frac{R_r'}{s}$ = $\frac{\text{Resistance of rotor referred to stator}}{\text{slip}}$

X_{ls} = Leakage reactance of stator

X_{lr}' = Leakage reactance of rotor referred to stator.

X_m = Magnetizing reactance.

$\bar{V}_{as}$ = Terminal voltage phasor across phase a

$\bar{I}_{as}$ = Current entering phase a in the stator

$\bar{I}_{ar}'$ = Rotor current phasor "rotated" and referred to the stator.

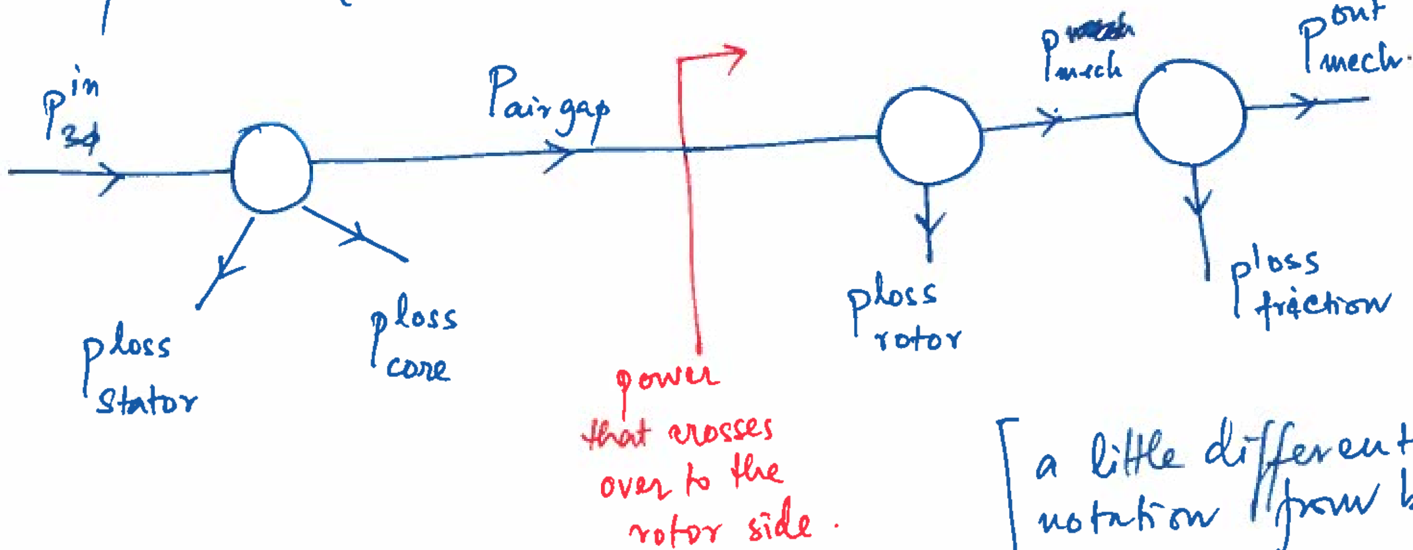
multiplied by $e^{j(\text{some angle})}$.

Agenda:

1. Power and torque calculations
2. Torque-slip curve.
3. Approximate circuit equivalent.

① Power and torque calculations.

$$P^{in}_{3\phi} = \mathrm{Re}\left\{3\, \bar{V}_{as}\, \bar{I}_{as}^{\,*}\right\}.$$

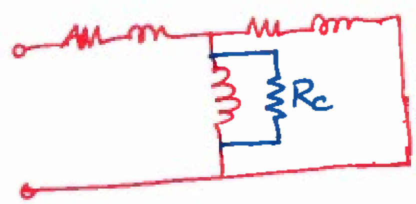


[a little different notation from book].

P^{loss}_{stator} (often written as P_{SCL}) $= 3 \cdot |\bar{I}_{as}|^2 \cdot R_s$

P^{loss}_{core} (often " " P_c)

due to hysteresis and eddy currents



Recall transformers!

P_{airgap} (often " " P_{AG}).

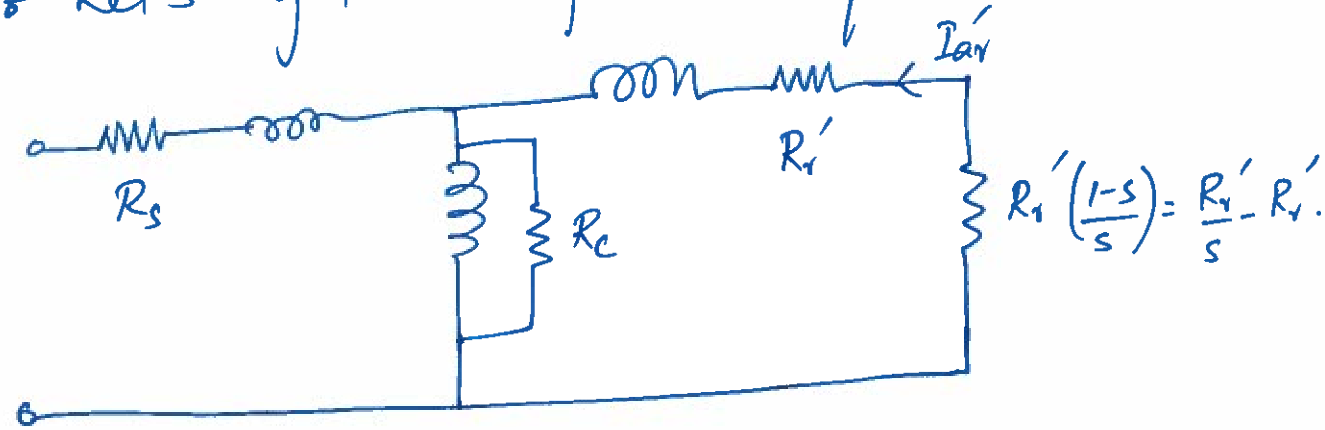
P^{loss}_{rotor} (often " " P_{RCL} or P_r) $= 3\, |\bar{I}_{ar}'|^2 \cdot R_r'$

$P^{loss}_{friction}$ (often " " P_{rot}) ~~unspecified directly~~.

P_{mech} (often " " P_m) = amount of mechanical power developed.

P^{out}_{mech} (often " " P_{out}) = useful shaft torque.

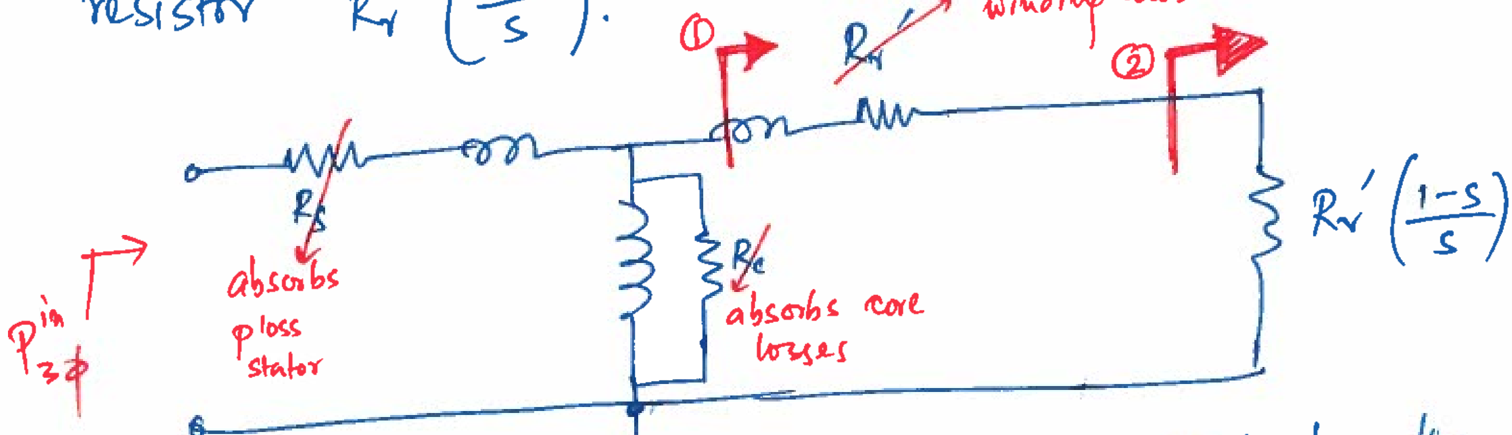
$P_{air\,gap}$: Let's get this from the equiv. ckt.



NOTICE :

$$\frac{R_r'}{s} = R_r' + \left(\frac{R_r'}{s} - R_r'\right)$$

Now, look at the power dissipated across the resistor $R_r'\left(\frac{1-s}{s}\right)$.



Q. What's the power absorbed by the circuit to the right of the arrows at ① and ② ?

① $= P^{in}_{3\phi} - P^{loss}_{stator} - P^{loss}_{core}$

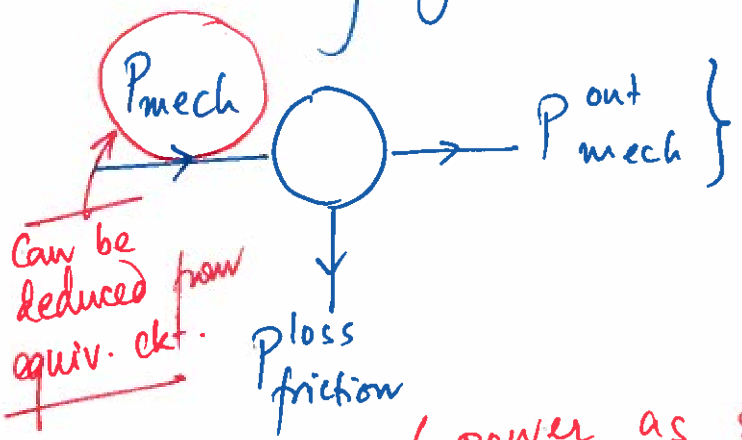
$= P_{air\text{-}gap}$

② $P_{air\,gap} - P^{loss}_{rotor}$

$= P_{mech}$.

If there's no frictional loss, $P_{mech} = P^{out}_{mech}$.

Circuit only gives us info about P_{mech}.



~~useful shaft torque.~~ "useful" mechanical power ..... or useful shaft power.

* P_{mech} = (power as seen from the arrow marked with ②) $= 3 \cdot |\bar{I}_{ar}'|^2 \cdot R_r' \left(\frac{1-s}{s}\right)$.

* $P_{air\text{-}gap}$ = (power as seen from arrow marked as ①) $= 3 \, |\bar{I}_{ar}'|^2 \cdot \frac{R_r'}{s}$.

$\therefore$ * $\boxed{P_{air\text{-}gap} = \frac{P_{mech}}{1-s}}$

How to compute T^e?

* $\boxed{T^e \cdot \omega_m = P_{mech}}$, and $\omega_m = (1-s)\,\omega_s$.

~~Also~~ $\therefore T^e = \frac{P_{mech}}{\omega_m} = \frac{P_{mech}}{(1-s)\,\omega_s}$

Also ...... $P_{mech} = P_{air\text{-}gap} \cdot (1-s)$

$$\Longrightarrow T^e = \frac{P_{air\text{-}gap}}{\omega_s}.$$

All calculations done so far are for 2-pole machines ...

For p-pole machines, $\boxed{\frac{p}{2}\omega_m = (1-s)\,\omega_s}$. or $\boxed{\omega_s - \omega_r = \frac{p}{2}\omega_m}$.

! $T^e \cdot \omega_m = P_{mech}$ always holds.

Only the relation between ω_m and ω_s changes when $p > 2$.

Example: A 3ϕ- induction motor has 4-poles, runs on 60Hz, 120V (line-neutral) balanced 3ϕ input. Its characteristic quantities referred to the stator are given by

$$R_s = 0,\quad X_{ls} = 0,\quad X_m = 40\Omega,\quad X_{lr}' = 1\Omega,\quad R_r' = 1.3\Omega,$$

Neglect core and frictional losses. Suppose the rotor speed is given by 1720 RPM.

Find s, ω_r, $|I_{ar}'|$, P_{mech}^{out}, T^e, $P_{3\phi}^{in}$, $P_{air\text{-}gap}$, power factor of the motor, efficiency of the motor.

In tests, we shall provide you with the terms and not just symbols.

Let's dissect the problem statement:

$60\,Hz \Rightarrow \omega_s = 2\pi \cdot 60$ rad/s
$= 377$ rad/s.

$120\,V$ (line-neutral) $\Rightarrow |\bar{V}_{as}| = 120\,V$.

You can always assume one angle in the ckt as a reference. Choose $\angle \bar{V}_{as} = 0$

$\therefore \bar{V}_{as} = 120 \angle 0$ Volts.

4-poles $\Rightarrow p = 4$. $\Rightarrow \omega_m \cdot \frac{p}{2} = \omega_s - \omega_r = (1-s)\omega_s$.

Neglect core/frictional losses. $R_c = 0$, $P_{\text{friction}}^{\text{loss}} = 0$.

Rotor speed is 1720 $\underbrace{RPM}_{\text{rotation per minute}}$

$$\therefore \omega_m = \frac{1720 \times 2\pi \text{ rad}}{60\,s}$$ why? 1 rotation $= 2\pi$ rad, 1 minute $= 60\,s$.

$= 180.1$ rad/s.

Let's embark on solving the problem now!

$\underbrace{\omega_s}_{\text{we know}} - \omega_r = \underbrace{\omega_m}_{\text{we know}} \cdot \overbrace{\frac{p}{2}}^{\text{we know.}} \Rightarrow \omega_r = \omega_s - \omega_m \cdot p/2$

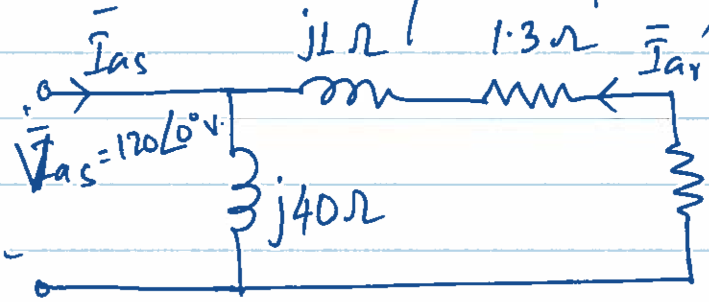
$= 377 - 180.1 \times \frac{4}{2}$ rad

$= 16.8$ rad/s.

$$s\omega_s = \omega_r \Rightarrow s = \frac{\omega_r}{\omega_s} = \frac{16.8}{377} = 0.045$$

$\boxed{s \text{ has no units!}}$

Now, let's draw equivalent circuit with the characteristic quantities as specified.



$$R_r'\left(\frac{1-s}{s}\right) = 1.3 \times \left(\frac{1-0.045}{0.045}\right)\Omega$$

$$= 27.6\Omega.$$

- $$\bar{I}_{ar}' = \frac{-\bar{V}_{as}}{j1\Omega + 1.3\Omega + 27.6\Omega}$$

$$\Rightarrow |\bar{I}_{ar}'| = \frac{120}{|j1 + 1.3 + 27.6|} \text{ Amps} = 4.1 \text{ Amps.}$$

- $$P_{mech}^{out} = P_{mech} = \text{power spent in } R_r'\left(\frac{1-s}{s}\right) = 3 \cdot \overbrace{|\bar{I}_{ar}'|}^{= 4.1 \text{ Amps}}{}^2 \cdot \overbrace{R_r'\left(\frac{1-s}{s}\right)}^{= 27.6\Omega}$$

$$= 1.43 \text{ kW}$$

- $$P_{air\,gap} = 3 \cdot |\bar{I}_{ar}'|^2 \cdot \frac{R_r'}{s} = 3 \times 4.1^2 \times \frac{1.3}{0.045} \text{ W} = 1.49 \text{ kW.}$$

$$\bullet \quad P_{3\phi}^{in} = \underbrace{\cancel{P_{stator}^{loss}}}_{=0\ (\because R_s = 0)} + \overbrace{\cancel{P_{core}^{loss}}}^{=0\ (\text{given})} + P_{air\text{-}gap}.$$

$$= P_{air\text{-}gap} = 1.49 \text{ kW}.$$

$$\bullet \quad T^e = \frac{P_{mech}}{\omega_m} = \frac{1.43 \text{ kW}}{180.1 \text{ rad/s}} = 7.94 \text{ N-m}.$$

$$\bullet \quad \text{Efficiency} = \frac{P_{mech}^{out}}{P_{3\phi}^{in}} \times 100\% = \frac{1.43 \text{ kW}}{1.49 \text{ kW}} \times 100\%$$

$$\approx 96\%.$$

Oh! We forgot to compute the power factor!
To do that, we need the angle of $\bar{I}_{as}$.

$$\bar{I}_{as} = \frac{\bar{V}_{as}}{Z_{eq}}, \quad \text{where } Z_{eq} = j40 \parallel (j1 + 1.3 + 27.6)\Omega$$

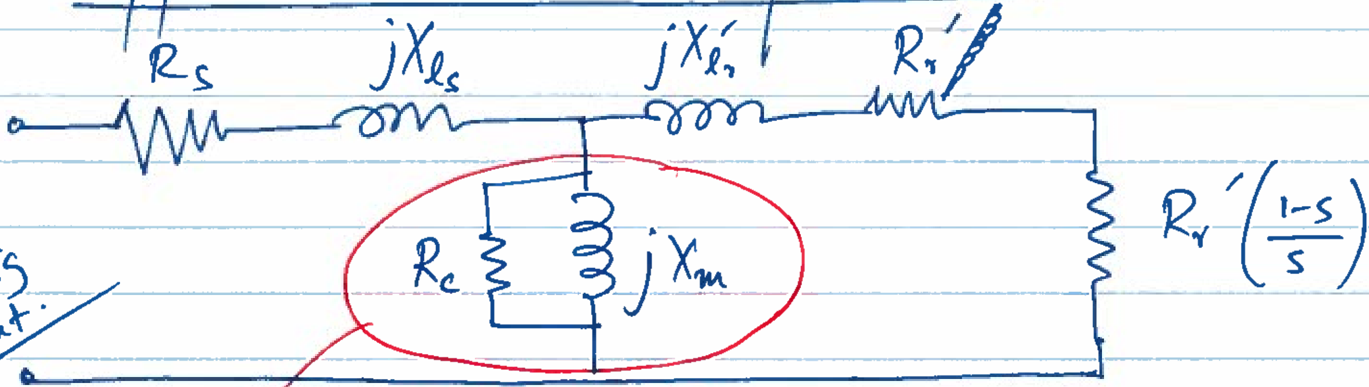
$$= (18.4 + j13.9)\Omega.$$

$$= 5.20 \angle -37.16°.$$

$$\text{power-factor} = \cos(37.16°) \approx 0.8 \text{ lagging}.$$

② Approximate circuit equivalent

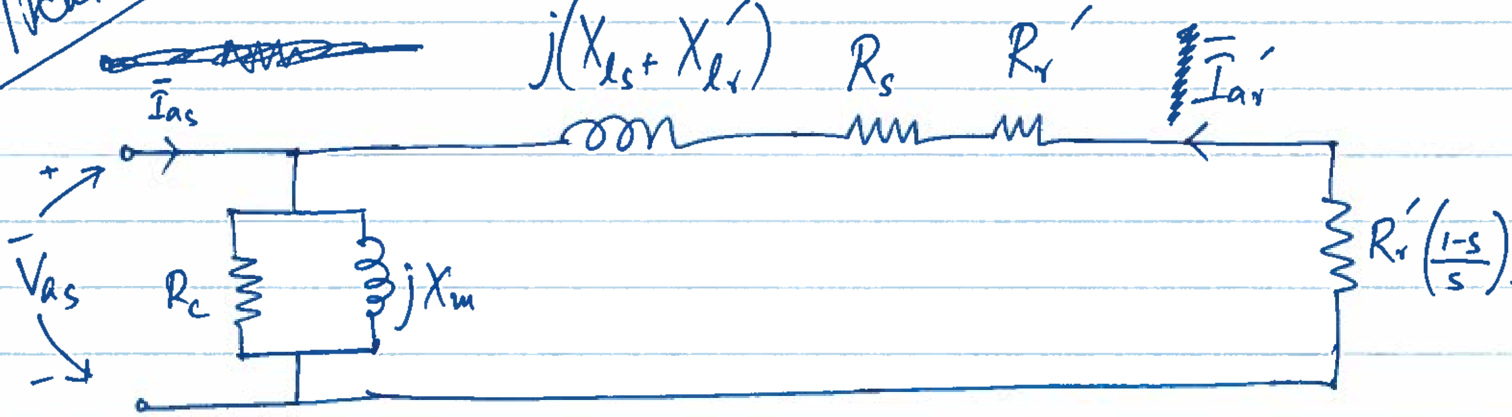
Orig ckt.



Remember the approx. ckt equivalent for a transformer?

Shift this block to the left of R_s, jX_{l_s}.

Approx. ckt.



(3)

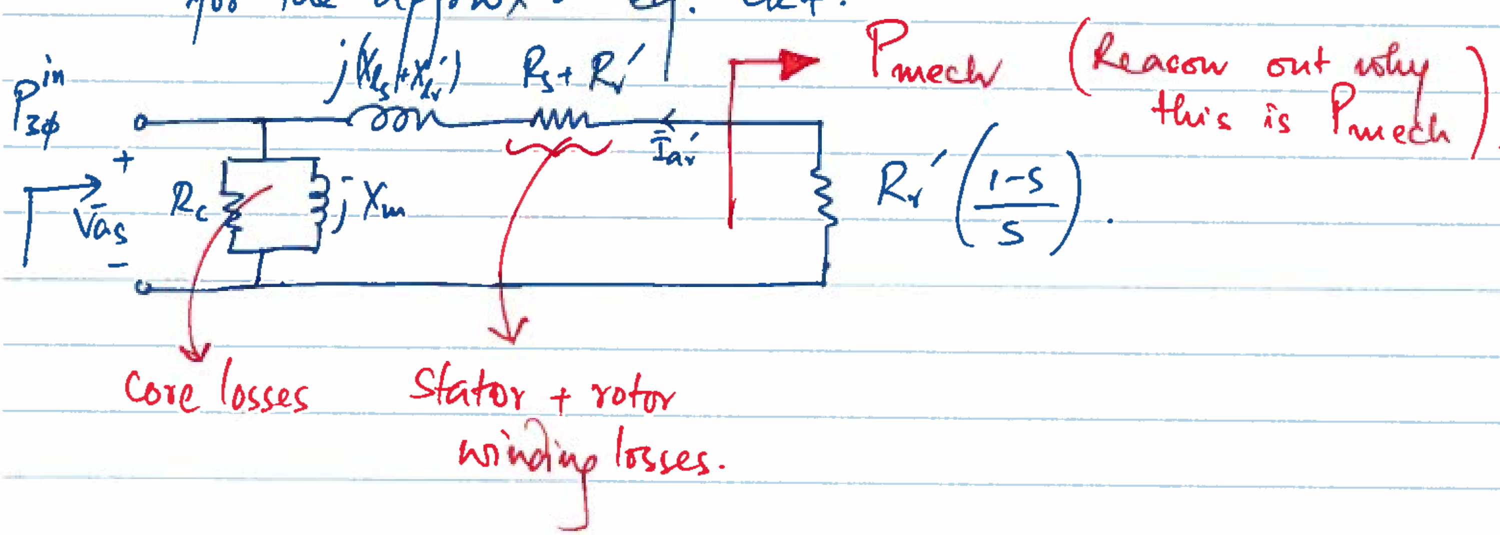
Torque-speed characteristics.

(speed → slip)

Q. How does T^e vary with slip? Sketch it!

Use the approximate equivalent circuit.

<u>Sol</u>^{n}: Let's use our logic to deduce what T^e is for the approx. eq. ckt.



$$P_{mech} = 3\cdot\left|\bar{I}_{ar}'\right|^2 \cdot R_r'\left(\frac{1-s}{s}\right).$$

write out $\bar{I}_{ar}$ in terms of $\bar{V}_{as}$ and impedances.

$$= 3R_r'\left(\frac{1-s}{s}\right)\left|\frac{\bar{V}_{as}}{\underbrace{jX_{ls}+jX_{lr}'}_{:=jX} + \underbrace{R_s + R_r' + R_r'\left(\frac{1-s}{s}\right)}_{=R_s+\frac{R_r'}{s}.}}\right|^2$$

$$= 3R_r'\cdot\frac{1-s}{s}\cdot\frac{\left|\bar{V}_{as}\right|^2}{X^2+\left(R_s+\frac{R_r'}{s}\right)^2}.$$

$$T^e = \frac{P_{mech}}{\omega_m}$$

$$\underbrace{\underbrace{\omega_s - \omega_r}_{= s\omega_s} = \frac{p}{2}\,\omega_m}$$

$$\Rightarrow \omega_m = (1-s)\,\omega_s \cdot \frac{2}{p}$$

$$= \frac{P_{mech}}{(1-s)\cdot\omega_s\cdot\frac{2}{p}}.$$

$$= 3\cdot R_r' \,\frac{\cancel{1-s}}{s}\cdot\frac{|\bar{V}_{as}|^2}{X^2 + \left(R_s + \frac{R_r'}{s}\right)^2}\cdot\frac{1}{\cancel{(1-s)}\cdot\omega_s\cdot\frac{2}{p}}$$

$$= \underbrace{\left(\frac{3p\cdot R_r'\cdot|V_{as}'|^2}{2\,\omega_s}\right)}_{\text{const.}}\cdot\left(\frac{1}{s\left[X^2 + \left(R_s + \frac{R_r'}{s}\right)^2\right]}\right).$$

Let's plot what this function looks like!

Use the following parameters:

$p = 4$, $|\bar{V}_{as}| = 120$, $\omega_s = 377$ rad/s.

$X_{ls} = 0.01\,\Omega$, $X_{lr}' = 1\,\Omega$,

$R_s = 0.01\,\Omega$, $R_r' = 1.3\,\Omega$.

Same values as in the example.

Only these are perturbed from the example.

Q. Can you find the slip s for which $T^e(s)$ is max? Also, find $\max\limits_{s \in \mathbb{R}} T^e(s)$.

Repeat the entire process for finding where T^e is min.

— We shall do this next class.

V.V. Imp.

Things to remember for induction machines.

- Equivalent circuit $\longrightarrow$ approx. equiv. circuit.
- Power considerations.
  - Remember: Circuit only computes P_{mech}, the mechanical power developed by the machine.

can be computed from ckt.

  - $\text{Power input electrical} \xrightarrow{} \text{Power through air gap} \longrightarrow \text{Mech. power developed.}$
    - Power input electrical $\to$ Stator winding losses, Core losses
    - Power through air gap $\to$ Rotor winding losses
  - $P_{mech} - P^{loss}_{friction} = P^{out}_{mech}$ } Useful output power.
- $\omega_s - \omega_r = \frac{P}{2} \cdot \omega_m$, and $\omega_r = s\,\omega_s$.
- $T^e = P_{mech} / \omega_m$.

Rest is manipulation!